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Toroidal Magnetic Spacecraft Shield Used to Deflect Energetic Charged Particles

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The use of magnetic fields to deflect energetic charged particles has been proposed as a means to protect astronauts from the harmful radiation encountered in space. These so-called active magnetic shields must provide a region of space that is protected from energetic particles below a given energy, while also maintaining a safe level of magnetic field strength within the shielded region. A toroidally shaped environment with circular coils of wire distributed on the surface of the spacecraft is shown to satisfy these requirements. Numerical techniques are used to demonstrate that particles below a given energy, including galactic cosmic rays, are completely shielded from a region inside the toroidal spacecraft. By choosing the appropriate amplitudes of the currents flowing in the circular coils, the magnetic field strength inside this region can also be made arbitrarily small inside the toroidal spacecraft. Although many practical issues must be addressed with this design, it has been demonstrated that it is indeed possible to construct a magnetic field suitable for protecting astronauts from galactic cosmic rays during long-duration manned missions.

Nomenclature

$\mathbf{A}$	=	vector potential of the magnetic field
A_ϕ	=	ϕ component of the vector potential
a	=	radius of coil
B	=	magnetic flux density
$\mathbf{B}_{\mathrm{in}}$	=	vector magnetic flux density inside boundary
$\mathbf{B}_{\mathrm{out}}$	=	vector magnetic flux density outside boundary
C_{st}	=	Störmer length
c	=	speed of light
I	=	electrical current in coil
$\mathbf{J}$	=	surface current density
$\mathcal{L}$	=	relativistic Lagrangian of a particle
L	=	inductance
M	=	magnitude of magnetic dipole moment
$\mathbf{M}$	=	vector magnetic dipole moment
m_0	=	rest mass of particle
n	=	number of turns of wire
$\hat{n}$	=	unit vector normal to boundary
p_ϕ	=	ϕ component of the generalized momentum
q	=	electrical charge of the particle
R	=	magnetic rigidity of a particle
r,θ,ϕ	=	radius and angles in standard polar coordinates
$\dot{r},\dot{\theta},\dot{\phi}$	=	generalized velocities associated with the r,θ,ϕ coordinates
r_b	=	radius of circular approximation to cross section of torus
r_c	=	radius to center of circular cross section approximation
s	=	distance along magnetic field line
$\mathbf{v}$	=	vector velocity of particle
v	=	speed of particle
x,y,z	=	standard rectangular coordinates
α	=	angle around spacecraft cross section
γ	=	Lorentz relativistic correction factor
λ	=	magnetic latitude
μ_0	=	permeability of free space
ρ	=	radius in standard cylindrical coordinates
$\hat{\phi}$	=	ϕ unit vector

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I. Introduction

THE need to protect astronauts from the harmful effects of space radiation in the form of energetic particles is a problem that must be addressed if manned missions of extended duration, such as those to Mars, are to be a reality. The most harmful of these particles are solar energetic particles (SEPs) resulting from large solar flares and galactic cosmic rays (GCRs) originating from outside our solar system. Although the particle flux of each population falls off steeply with increasing energy, SEPs are typically <100 MeV per nucleon and the GCR spectrum peaks around 1 GeV per nucleon before falling off [1–3]. Although there is considerable uncertainty in assessing the risks to astronauts from these energetic particles, it is generally agreed that particles with energies of 1–4 GeV per nucleon are the most damaging to humans [4–7].

Traditional techniques for protecting spacecraft from these forms of radiation typically involve a protective shield of material used to absorb the energy of incoming particles. Although these so-called passive shields can be effective at blocking particles with lower energies, the mass required for protecting against energetic particles such as GCRs becomes impractical for use in spacecraft. In addition, higher energy particles colliding with a passive shield produce a cascade of lower energy particles that could be even more damaging to human cells than the original particles. For these reasons, an active shield which uses electric or magnetic fields to deflect energetic particles is a possible alternative for long-duration, manned missions. A review of active shields was performed by Sussingham et al. [8].

Several concepts for these so-called active shields have been proposed, including electrostatic shields [9,10] and magnetic shields, that either include an artificial plasma environment [11] or that do not [6,12–14]. Electrostatic shields are generally considered to be impractical for shielding GCR particles, due to the difficulties of generating a sufficiently high-voltage potential, and are too dangerous in situations of accidental discharge [13].

Magnetic shields can be roughly divided into those which rely solely on magnetic fields to deflect particles and those which have an artificial plasma environment that acts to inflate the existing magnetic field. The primary advantage of an artificial plasma environment is the claim that particles of a given energy are deflected using a much lower magnetic field strength than would be required using a magnetic field only [11]. There is, however, some question as to the effectiveness of such a device to shield particles, particularly along the magnetic field axis [15]. Whether such a concept works as an effective shield or not, the addition of an artificial plasma clearly represents an increase in complexity of the overall design that we avoid in further discussion.

Shields which rely strictly on static magnetic fields to deflect charged particles via the Lorentz force can be further categorized as either confined or deployed, depending on the configuration of the current carrying wires (or coils) relative to the habitable portion of the spacecraft. Deployed magnetic shields rely on a configuration of coils (the simplest being a circular coil of radius a) that are located at large distances from the spacecraft. The main advantage of a deployed shield is that the amount of electrical current I needed to sustain a given magnetic dipole moment magnitude ($M = I\pi a^2$) decreases as the size of the coil increases [12,16–18]. It has been shown, however, that the shielding capacity of such coils is also reduced significantly, to the extent that no shielding occurs in a region near the center of the coil [19,20].

Confined magnetic shields, on the other hand, are so named for the coils being located in close proximity to, or even as an integral part of, the spacecraft living area [6,14,21–23]. In these devices, it has been shown that, by using sufficiently large electrical currents, it is possible to shield the entire spacecraft from energetic particles. For some of these devices, however, the magnetic field strength produced by the coils, which is necessary to protect from GCR particles, is extraordinarily high (>10 T) in the vicinity that is being shielded. Although it is relatively unknown what the effects of high-strength static magnetic fields are on humans, there are many reports of individuals experiencing various neurological effects while in motion near magnetic resonance imaging (MRI) machines with magnetic field strengths <10 T [24,25].

A dilemma of sorts seems to exist with the design of confined magnetic spacecraft shields in that a high magnetic field strength is needed to deflect energetic GCR particles, yet a low field strength is desired for the safety of the spacecraft occupants. One solution to this problem is to confine the magnetic field to a relatively small region around the spacecraft [6,14]. We propose an alternate solution which satisfies both of these apparently contradictory design requirements. The device we propose consists of a toroidally shaped spacecraft, with a noncircular cross section in the ρ–z plane, where $\rho^2 = x^2 + y^2$ defines the polar coordinate (see Fig. 1). Circular wires (coils) are located around the exterior of the spacecraft, through which electrical current flows in the toroidal direction, providing the magnetic field necessary to deflect energetic particles. By adjusting the strength of the currents flowing in each coil, it is possible to create a magnetic field that is both large exterior to the spacecraft (thereby shielding the occupants) and arbitrarily small inside the spacecraft (thereby providing a nonhazardous environment).

The remainder of the paper is organized as follows: Sec. II examines the shielded region produced by a single coil of wire which provides the motivation for using a toroidally shaped spacecraft. The shielded regions are determined using two numerical techniques: the first involves numerical solutions of the generalized azimuthal angular momentum of a particle moving in a magnetic field, and the second a test-particle simulation in which particle trajectories are calculated. Section III introduces the shape of the toroidal spacecraft and the configuration of coils that allows the magnetic field to be reduced to zero inside the spacecraft. A discussion of the shielding capabilities of the toroidal spacecraft is presented in Sec. IV in addition to some of the many practical considerations for building such a spacecraft. Finally, a summary is given in Sec. V.

II. Shielded Region Due to Circular Coil

The motivation for the design of toroidally shaped spacecraft comes in part from the analysis of particle trajectories in the presence of magnetic fields. For certain configurations, the influence these magnetic fields impart on charged particles through the Lorentz force can lead to regions of space for which particles below a given energy do not have access or are forbidden. These regions of space are said to be shielded from such particles.

Derivations of these regions, similar to the one described here, have been presented before [26–28]. We begin a brief derivation with the relativistic Lagrangian of a particle of mass m_0 and charge q, moving with velocity $\mathbf{v}$ in a static magnetic field with vector potential $\mathbf{A}$ and in the absence of an electric field [29]. If the magnetic field is axially symmetric about the z axis, that is, $\mathbf{A} = A_\phi \hat{\phi}$ in polar coordinates, then

$$\mathcal{L} = -\gamma^{-1} m_0 c^2 + q A_\phi r \sin\theta \dot{\phi} \tag{1}$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorentz relativistic correction factor, c is the speed of light, r is the radial distance from the origin to the particle in spherical coordinates, θ is the angle from the z axis, $\dot{\phi}$ is the generalized velocity associated with the ϕ coordinate, and SI units are used throughout.

Because Eq. (1) is independent of ϕ, the Lagrangian equation of motion for the ϕ coordinate can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t} p_\phi = \frac{\partial \mathcal{L}}{\partial \phi} = 0 \tag{2}$$

and p_ϕ is therefore a constant of the motion given by

$$p_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = -m_0 c^2 \frac{\partial \gamma^{-1}}{\partial v} \frac{\partial v}{\partial \dot{\phi}} + q A_\phi \sin\theta \tag{3}$$

Using $v^2 = \dot{r}^2 + r^2\dot{\theta}^2 + r^2 \sin^2\theta \dot{\phi}^2$ to evaluate the partial derivatives in Eq. (3), the azimuthal component of the generalized momentum can be written as

$$p_\phi = \gamma m_0 v_\phi r \sin\theta + q A_\phi r \sin\theta \tag{4}$$

A closed-form solution to Eq. (4) has been shown to exist for only a few special magnetic field configurations. Carl Störmer showed

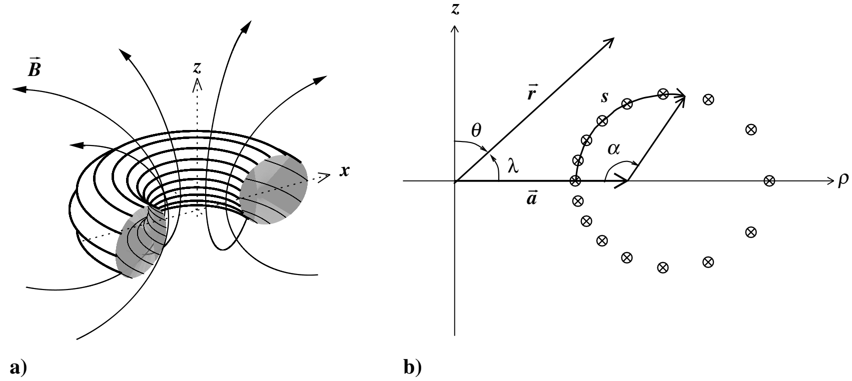

Fig. 1 Proposed magnetic spacecraft shield: a) cutaway view, b) cross section.

that such a solution existed for a pure dipole magnetic field [30]. Inserting the vector potential of a pure dipole $A_\phi = -(\mu_0/4\pi)M\sin\theta/r^2$, with dipole moment $\mathbf{M} = M\hat{z}$ into Eq. (4), the equation describing the shielded region of a magnetic dipole located at the origin is given by

$$r = \sqrt{\frac{Mq\mu_0}{4\pi\gamma m_0 v}}\frac{\cos^2\lambda}{1+\sqrt{1+\cos^3\lambda}} \tag{5}$$

where λ is the magnetic latitude ($\pi/2 - \theta$, see Fig. 1). The factor in front of the radical in Eq. (5) has units of length and is often referred to as the Störmer length, given by

$$C_{\rm st} \equiv \sqrt{\frac{Mq\mu_0}{4\pi\gamma m_0 v}} = \sqrt{\frac{M}{R}\frac{\mu_0}{4\pi}} \tag{6}$$

which can be written in terms of the magnitude of the dipole moment M and the so-called rigidity of the particle $R \equiv \gamma m_0 v/q$.

Equation (5) describes a toroidal shielded region around the origin. In the equatorial plane ($\lambda = 0°$), the region extends a distance approximately $0.4C_{\rm st}$. Along the axis of the dipole ($\lambda = \pi/2$), however, the distance of this region is zero, that is, no shielding occurs for particles moving along the axis of the dipole. Regardless of the magnitude of the dipole moment M, it is therefore not possible to completely shield a region of any size at the origin from particles of any energy. For this reason, it is more sensible to build a spacecraft that is more congruent with the geometry of the shielded region, that is, toroidal. In this case, shielding particles that are traveling along trajectories near the axis of the dipole is not an issue, as they pass harmlessly through the center of the toroidal spacecraft.

Several confined magnetic shields of this design have been proposed [22,23,31]. In these cases, however, the dipole magnetic field is approximated by the field due to a coil of finite radius a with n turns of wire, each carrying a current of I. The magnetic field of such a coil provides shielding of the type described in Eq. (5) only when $a \ll C_{\rm st}$ [19,20].

Shielding also occurs for situations in which $a \geq C_{\rm st}$, however, not of the type described by Eq. (5), that is, the dipole approximation is not valid. To determine the shielded region due to a coil of wire, Eq. (4) must be solved using the vector potential A_ϕ of the coil. Levy [27] shows that, although it is not possible to write a closed-form solution to this equation, a numerical solution is possible.

Figure 2 shows the shielded regions calculated using this numerical technique for three different idealized coils of radii: $a = 3.415$, 9.115, and 14.35 m. The location of the single coil is shown by the circular symbol with a cross, indicating that the current flows into the page, and the wire is assumed to have no thickness. Protons with energy 1 GeV or less are forbidden from the toroidal region, described by a dashed line in Fig. 2, around the coil. The total current required to shield such particles in each case is $nI = 6.451 \times 10^8$, 1.688×10^8, and 1.071×10^8 A for Figs. 2a–2c, respectively. For reference, the strength of the magnetic field in each case is indicated by the logarithmic gray scale contours.

Also shown in Fig. 2 are the positions of representative 1 GeV H^+ (protons) test particles at their closest approach to the coil. Particle trajectories are calculated using a numerical technique described by Shepherd and Kress [19]. Briefly, the coupled set of differential equations describing the motion of a particle of given energy, mass, and charge in the presence of an external magnetic field are solved using a standard Runge–Kutta fourth-order method. To accurately resolve the particle trajectories, an adaptive time-step is used by adjusting the step size to be 0.1% of the instantaneous gyroperiod of the particle. A maximum step size of 1 μs is used, but no lower bound on the minimum step size is imposed. Although this numerical technique does not explicitly conserve adiabatic invariants, it has been demonstrated to accurately resolve shielded regions and geomagnetic cutoffs due to static magnetic fields [19,20,32,33].

As shown in Fig. 2, agreement between the boundaries determined using two entirely different numerical techniques is excellent. Although many test particles come within a small fraction of 1 m to

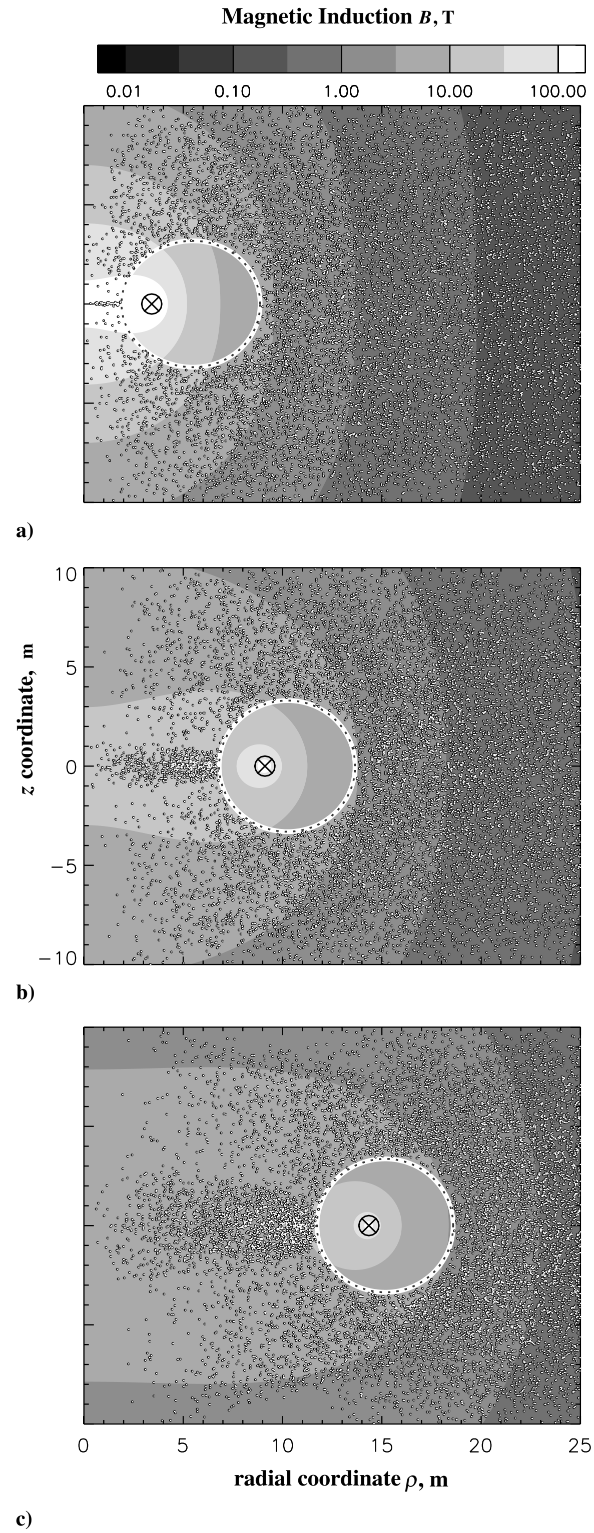

Fig. 2 Shielded regions determined using two different numerical techniques for single-loop coils of three different radii.

the boundary, no particle penetrates this region. For these simulations, particles are initiated at random positions uniformly distributed on a sphere of radius 1 km. To sample the possible trajectories that could penetrate the region near the coil, particles are initiated with velocities according to their energy (1 GeV) and directed toward another random point on a sphere of radius 500 m centered at the origin. The resulting sampling of test-particle trajectories is not intended to represent a true distribution function, but rather an appropriate sample that reveals any shielded regions located near the coils.

Figure 2 shows that a spacecraft enclosing the coil would contain a shielded region near the coil to which particles of a given energy do not have access. Toroidal spacecraft of this design have been proposed [23]. The main problem with these designs is that the strength of the magnetic field required to deflect GCR particles is very large. This field is strongest near the coil, which is enclosed within the spacecraft, thereby exposing the occupants to very large static magnetic fields. The field strength inside the shielded regions shown in Fig. 2 is >3 T and a minimum on the shielded surface itself. Moving away from the boundary toward the location of the coil, the field becomes unbounded.

Documented studies show exposure to static magnetic fields as low as ~ 0.1 T can affect blood flow around the heart, however, the physiological consequences of this interaction are unclear [34]. Although it is likely that long-term exposure to static magnetic fields would pose a serious health risk to astronauts, no studies of long-term exposure exist. The best available data on health risks associated with exposure to static magnetic fields come from laboratory and epidemiological studies involving magnetic resonance imaging and spectroscopy technicians. Although no threshold has been set for continuous exposure to static magnetic fields, the National Radiological Protection Board (NRPB) and other international agencies have set occupational guidelines for working with MRI in the form of time-weighted-average field exposure of 0.20 T per 8 h workday [24]. Although the effects of constant exposure to such fields over long durations is unknown, extrapolation of the NRPB recommendations would suggest that the magnetic field strength inside the spacecraft should not exceed a level of ~ 70 mT.

III. Magnetic Field Cancellation

For the example of a toroidally shaped spacecraft, which is suggested by the shielded regions shown in Fig. 2, it is therefore desired to reduce the field inside the torus (the shielded region), while leaving the field external to the spacecraft unchanged. Reduction of the magnetic field can be accomplished by adding additional coils in such a configuration that the magnetic fields oppose the field of the main coil, thereby canceling the field in the desired region. A relatively simple solution is to add a single coil that is coplanar to the main coil but with a smaller radius. The strength of the current in the secondary coil is chosen so that the superposition of the fields is zero on a circular path between the two coils [35].

Although this particular configuration is relatively simple and reduces the field in the region between the two coils, it also affects the fields out of the plane of the coils. Particles penetrating the region of space near the coils can have very complicated three-dimensional trajectories. Shielding of these particles is achieved by the detailed magnetic field near the coils, rather than by the overall magnetic moment [19]. Reducing the field in this manner can, therefore, also have the effect of reducing the shielding capacity for some particle trajectories. It is possible, in principle, to reduce the field inside the torus to exactly zero, everywhere, without affecting the field external to the spacecraft.

To show how such a configuration of coils is possible, we first determine the cross-sectional shape of the torus by computing a magnetic field line of the single coil shown in Fig. 2b. Figure 3 is a reproduction of Fig. 2b with the addition of four magnetic field lines computed from starting positions of $z = 0$ and $\rho = 2$, 4, 6, and 7.342 m. The latter field line (indicated by a thicker line) is chosen to represent a region that extends 5.0 m in the vertical z direction and is centered at $\rho = 10.0$ m. These dimensions represent a possible spacecraft design that would provide adequate space inside the torus for habitation.

The cross-sectional shape of the spacecraft is chosen to correspond to the shape of the magnetic field line represented by the thicker line in Fig. 3. In this configuration, the magnetic field, by definition, is tangential everywhere to the surface of the spacecraft. The magnetic field inside the spacecraft can be made to be exactly zero everywhere by specifying an appropriate surface current density $\mathbf{J}$ that flows along the outer surface of the spacecraft in the toroidal direction $\hat{\phi}$.

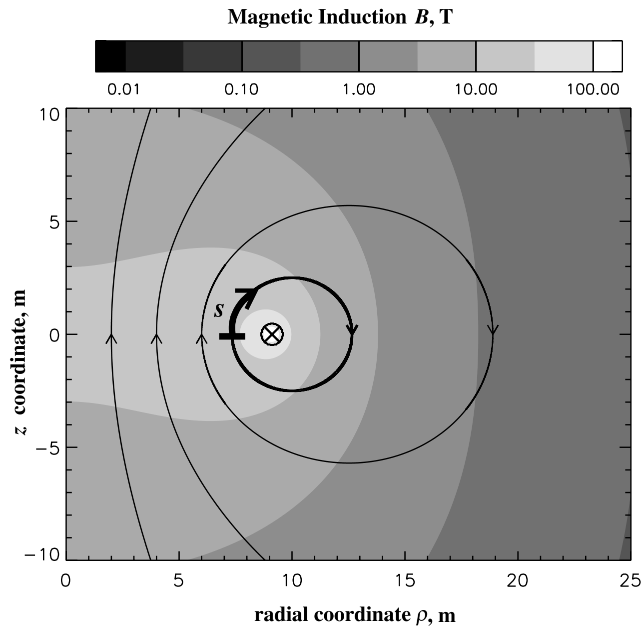

Fig. 3 Magnetic field lines of a single-loop coil that determine the cross section of the spacecraft.

The surface current density $\mathbf{J}$ is determined by invoking Ampere's Law at the boundary of the spacecraft. In this case, the jump in the tangential magnetic field at a boundary is proportional to the surface current density flowing along the boundary [36], or

$$\hat{n} \times (\mathbf{B}_{\text{out}} - \mathbf{B}_{\text{in}}) = \mu_0 \mathbf{J} \tag{7}$$

where $\hat{n}$ is normal to the magnetic field or spacecraft surface, $\mathbf{B}_{\text{in}}$ and $\mathbf{B}_{\text{out}}$ are the magnetic field vectors on the inner and outer edge of the spacecraft surface, and $\mathbf{J}$ is the surface current density in amperes per meter flowing in the $\hat{\phi}$ direction along the surface of the spacecraft.

By choosing the magnetic field to be zero on the inner edge of the boundary, $\mathbf{B}_{\text{in}} = 0$, Eq. (7) can be solved for the surface current density that is needed to produce the desired magnetic field:

$$\mathbf{J} = \frac{\mathbf{B}_{\text{out}}}{\mu_0} \tag{8}$$

Using the magnetic field resulting from the single coil along the spacecraft surface for $\mathbf{B}_{\text{out}}$, Eq. (8) gives the surface current density which produces the same magnetic field outside the spacecraft as the single coil, but the field inside the spacecraft is zero everywhere. Such a magnetic field provides shielding against particles below a certain energy without the associated high field strengths within the spacecraft that are associated with many confined magnetic shields.

The solid line in Fig. 4 shows the current density J as a function of distance along the field line that is needed to produce the desired fields for the configuration shown in Fig. 3. As shown in Figs. 1 and 3, the distance along the field line is given by s, measured from the location on the spacecraft surface closest to the origin. The total current J, integrated around the entire field line, is within 0.1% of the value determined for the single coil, 1.688×10^8 A.

The practical issue of controlling the distribution of current that flows along the surface of the spacecraft can be achieved by approximating the current density J with discrete wires or coils located around the surface of the spacecraft. The current flowing in each coil is determined by integrating the current density along the line segment (in the s direction) that extends half the distance to the adjacent coils. The dots in Fig. 4 indicate the positions of 32 circular coils distributed around the surface of the spacecraft and the current required in each coil. The 32 coils are distributed uniformly in the angle α around the surface (see Fig. 1). The total current in the 32 coils is again within 0.1% of the current in the single coil shown in Fig. 3.

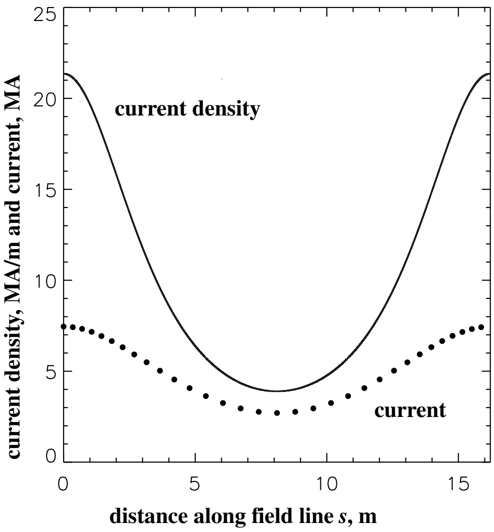

Fig. 4 Surface current density and current required to cancel magnetic field inside spacecraft.

As expected, the discrete approximation to the current density J leads to imperfect cancellation of the magnetic field inside the torus, and the approximation improves by using more coils. Figure 5 shows the magnetic field in the vicinity of the spacecraft that results from 8-, 16-, 32-, and 64-coil approximations. The magnitude of the magnetic field is represented in Fig. 5 by the logarithmic gray scale. Locations of the coils are indicated by the circular symbol containing a cross, indicating the current flowing into the page. The magnitude of the current in each coil varies according to the curves in Fig. 4.

As shown in Fig. 6, the field is not exactly zero everywhere inside the torus for the various approximations shown. The field, however, is significantly reduced from that of the single coil shown in Fig. 2b. The reduction obtained with the 8-coil approximation is the least accurate of the four cases shown in Fig. 5, however, even for this relatively poor approximation, the field is less than 2 T everywhere within a small fraction of a meter inside the spacecraft. Recall that, in the case of the single coil, the field was greater than 3 T at the boundary and increased without limit for locations approaching the coil. The cancellation improves with each doubling of the number of coils. Near complete cancellation is achieved for the 64-coil approximation shown in Fig. 5d. The field strength in this case is less

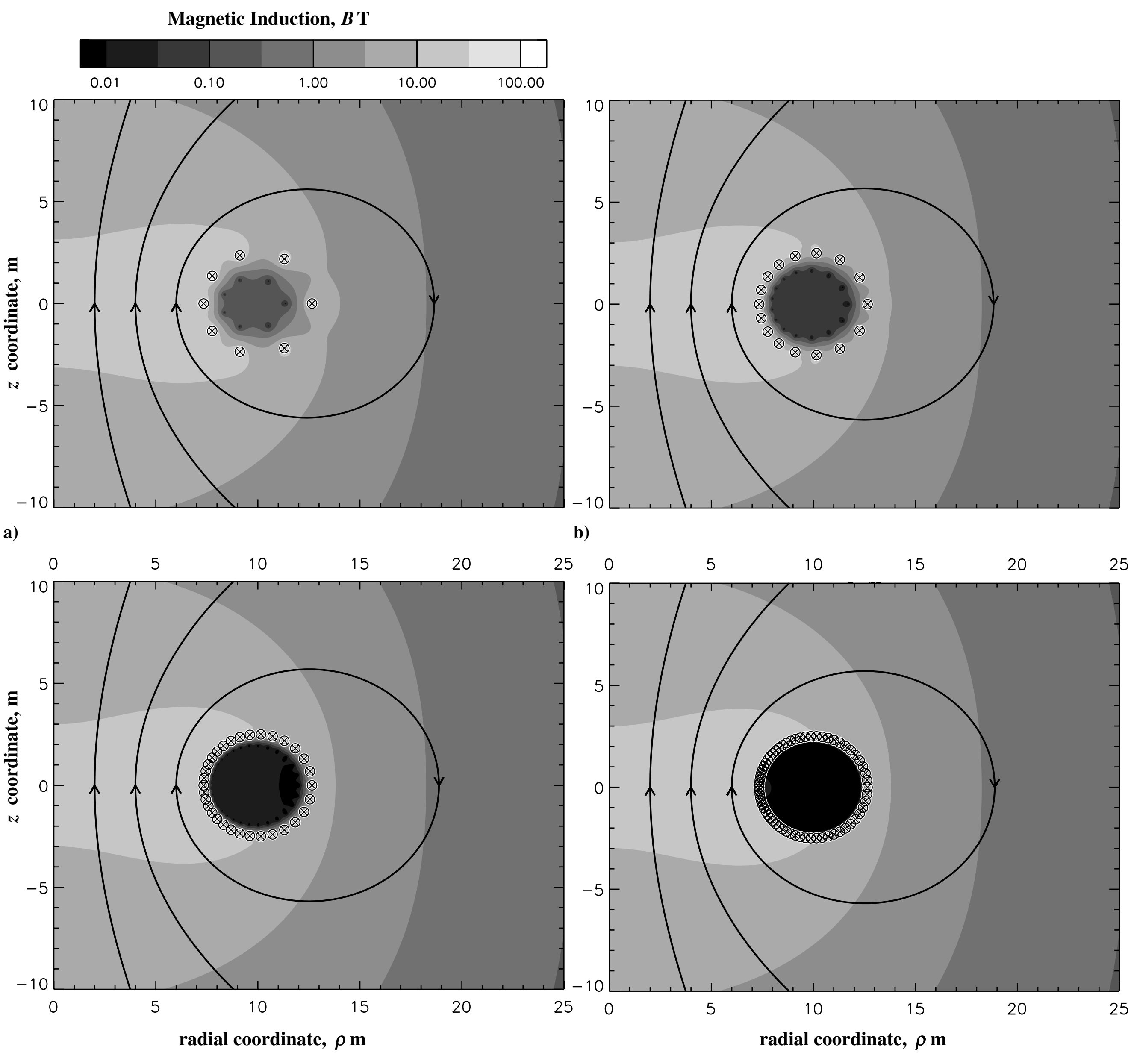

Fig. 5 Magnetic field cancellation for a) 8-, b) 16-, c) 32-, and d) 64-coil approximations to Eq. (8).

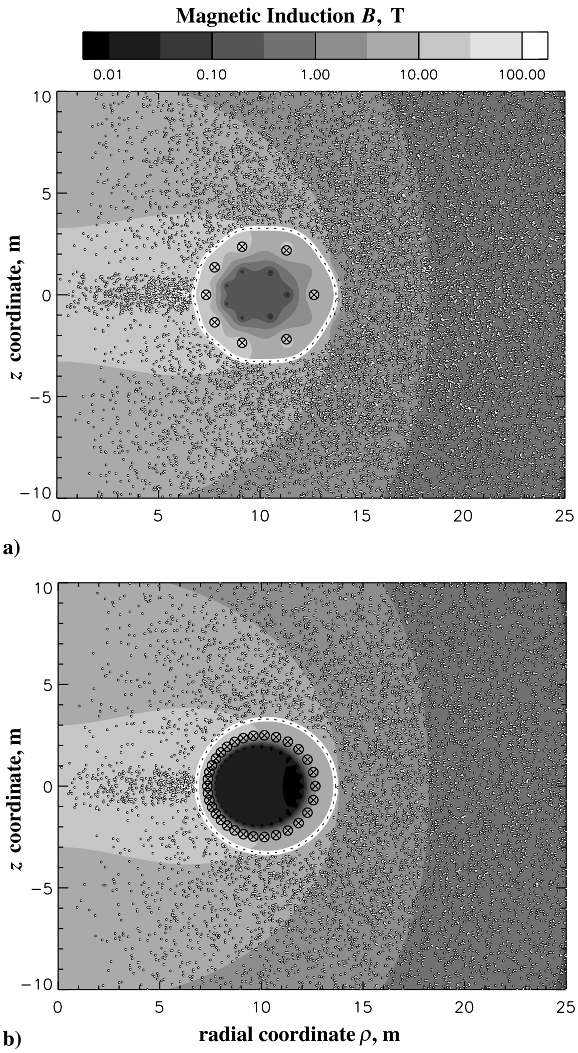

Fig. 6 Shielded regions computed using the full vector potential A_ϕ of the a) 8- and b) 32-coil configurations.

than 0.01 T at all locations inside the torus that are at least a distance of ~0.3 m from the interior surface of the spacecraft. As the number of coils approaches infinity, the cancellation becomes exact and the field everywhere inside the toroidal volume comprising the spacecraft decreases to zero.

IV. Discussion

The discrete nature of the approximation to Eq. (8) is evident in the magnetic field produced by the multiple-coil design (torus), particularly with the 8-coil example shown in Fig. 5a. In this example, the differences between the magnetic fields of the torus and the single coil are most apparent. At large distances from the surface of the spacecraft, the field from the multiple coils approximates the field from the single coils to very good agreement. In a region surrounding the coils, which extends to a distance roughly equal to the separation between the coils, the agreement breaks down.

The implication is that, for a shielded region, which extends beyond the surface of the spacecraft by a distance that is larger than the separation between the coils, the field that any particle will experience along its trajectory will be equivalent to that of the single coil. For such cases, the shielded region for the multiple-coil design can be obtained by solving Eq. (4) using the much simpler form of the vector potential A_ϕ for the single coil. Several studies use this approximation to describe the regions shielded by multiple-coil designs [27,37].

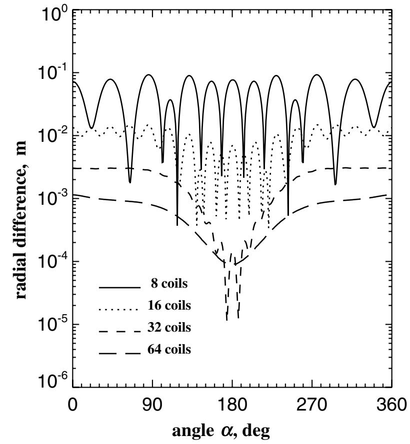

Fig. 7 Radial differences between shielded regions for the single-coil approximation and the multiple-coil configuration.

For situations in which particle trajectories approach the surface of the spacecraft within a distance approximately equal to the separation between coils, the particle will experience a different magnetic field from the single coil, and differences in the shielded regions will therefore exist. It is possible in such situations to solve Eq. (1) using the full vector potential A_ϕ for the multiple coils. Figure 6 shows the shielded regions calculated using the multiple-coil vector potential for the 8- and 32-coil examples shown in Figs. 5a and 5c. As with the other examples, the corresponding test-particle simulations show excellent agreement with the numerically determined shielded regions.

For these examples, the total current was chosen such that the shielded region for a 1 GeV proton would be 1 m from the outermost point of the single-coil configuration. Because of the proximity of the shielded region to the surface of the spacecraft, small differences are visible between the single- and multiple-coil regions shown in Figs. 2c and 6. To better quantify the single-coil approximation, Fig. 7 shows the radial difference between the boundary determined using multiple coils and the single coil as a function of the angle α. In general, Fig. 7 shows that the shielded region of multiple coils approaches that of the single coil in a roughly linear fashion as the number of coils increases.

Shielded regions of larger or smaller dimensions may be determined by increasing or reducing the current in each coil proportionately. It is possible to reduce the current in the single coil to the extent that a portion or all of the shielded region is contained inside the surface of the desired spacecraft. In such cases, particles have access to the interior of the spacecraft where the magnetic field is very low. The shielded region of the single wire is no longer a valid approximation in these situations. Shielded regions do exist under these conditions, however, the geometry of the region or regions due to the multiple coils can be rather complex.

Although it has been demonstrated that a magnetic field configuration can be constructed such that a region of very low magnetic field is contained entirely within a region for which particles below a given energy are forbidden access, there remain numerous practical issues for implementing such a design. Foremost is the large current required to shield GCR particles. For the examples shown in Fig. 2, the current required to shield 1 GeV protons from the toroidal spacecraft are $>10^8$ A. It is expected that superconducting coils would be required to minimize the significant electrical resistance and dissipation that is expected for such large currents. The magnitude of the required current can be somewhat reduced by using multiple turns of wire in each coil and increasing the number of coils used to generate the magnetic field. Even with these measures, however, it is unlikely that such large currents can be achieved with modern superconducting technology. Significant advancements in this area will be needed before this limitation can be overcome.

Using multiple coils and multiple turns of wire has the advantage of reducing the current required in each wire, however, it also adds

Table 1 Energy stored in coils

r_c, m	L, μH	I, MA	E, GJ	E, MW · h
5.0	4.87	645	1013	281
10.0	18.5	169	263	73
15.0	35.4	107	203	56

weight to the spacecraft. Superconducting coils would also require additional cooling apparatus and perhaps structural reinforcement, all of which adds further weight to the spacecraft. Although it is difficult to estimate how much additional weight is necessary for such a design, Levy [21] showed that the weight-reducing benefit of magnetic shields relative to passive shields was most significant for GCR particles. An analysis of the weight requirements depends on the details of how the coils are designed, but is necessary to ensure that weight reduction is significant enough to warrant the added complexity of a magnetic shield.

Another practical concern about the feasibility of such a device is the amount of energy required to power the shield. The energy stored in the magnetic field of an inductor is given by $E = LI^2/2$, where L is the inductance of the device and I is the total current flowing through the inductor. Whereas an accurate determination of this energy requires detailed calculations of magnetic flux surfaces for the toroidal coils, an approximate value can be obtained by assuming the total current I flows uniformly on the outer surface of a hollow conductor with a circular cross section of radius r_b. The shape of the hollow conductor is approximately that of the spacecraft surface. The inductance of such a device is given by [38]

$$L \approx \mu_0 r_c \left[\ell n \left(\frac{8 r_c}{r_b} \right) - 2 \right] \tag{9}$$

where r_c is the radial distance to the approximate center of the circular cross section. Using values for r_c in Eq. (9) and the corresponding current I from the three different shielded regions shown in Fig. 2, the approximate inductance and energy are calculated and shown in Table 1. A value of $r_b = 2.5$ m was used in each case. The large energy required to power the shield is a significant drawback of this design, in addition to the dangers associated with quenching of the coils.

There are other practical considerations that must be addressed, such as the stresses the coils induce on each other and on the spacecraft. Because the current in each coil flows in the same direction, it is expected that an overall force is directed inward from the surface of the spacecraft. Additional structural reinforcement of the spacecraft could be required to withstand these forces. Furthermore, in these examples, it was assumed that no magnetically permeable materials were present in calculating the magnetic fields. The presence of such materials would alter the resulting magnetic fields, perhaps significantly. A more detailed analysis of the effect of using magnetically permeable materials in the spacecraft would be required.

It is clear that there are numerous challenges to building a magnetic shield capable of protecting astronauts from GCR particles. Our intent is not to address all of these practical considerations, but rather to demonstrate that it is possible to construct a magnetic field, at least in principle, that is both strong enough to deflect incident GCR particles and, at the same time, low enough interior to the spacecraft so as not to cause undesirable physiological effects on the inhabitants. The toroidal design described here accomplishes these apparently contradictory requirements.

V. Conclusions

We have proposed a novel concept for a magnetic spacecraft shield which has the desired features of effectively shielding energetic particles from a toroidal region of space, while also maintaining a low (near zero) magnetic field in this region. The shape of the spacecraft is that of a torus with a cross section that is determined by the shape made by a magnetic field line resulting from a circular coil of wire contained within the torus. By making the spacecraft conform to this special shape, it is relatively easy to distribute electrical current in wires around the surface of the spacecraft such that the magnetic field inside is virtually eliminated. The resulting magnetic field can be designed to completely shield particles below a given energy, including GCR particles, from a region containing the spacecraft.

The boundaries of shielded regions are determined for several configurations using two different numerical techniques. The first method involves numerically solving an equation that describes a constant of the motion of charged particles in a magnetic field [27], including cases where the full vector potential due to multiple coils is required. The second method involves numerical solution of test-particle trajectories in the same magnetic field. Excellent agreement between boundaries determined using both methods is shown to occur.

Toroidal spacecraft have been previously suggested and there are advantages to such a design [21–23,35,37,39,40]. Principle among these advantages is moving the habitation region away from a centralized region near the origin. Magnetic shields located near the origin of the magnetic field all suffer from the serious flaw of being unable to shield particles along the main axis of the field, thereby allowing particles from some directions to penetrate the spacecraft [20]. No such difficulties are encountered with toroidal spacecraft, as particles approaching along the axis of the field pass harmlessly through the center of the torus.

Although it has been demonstrated that a magnetic field configuration is possible, at least in principle, that is strong enough to effectively shield GCR particles and, at the same time, weak enough inside the spacecraft so as not to induce a harmful environment to astronauts, there are many practical concerns that must be addressed with this design. Foremost among these issues is the large electrical current ($>10^8$ A) and energy ($>10^{11}$ J) that is required to provide a magnetic field capable of shielding GCR particles. Whether it is possible to safely and practically generate a current of this magnitude, as well as other issues associated with currents of this magnitude, is a challenge that must be overcome for a magnetic spacecraft shield of this design to be considered and make extended-duration manned missions a reality.

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